Course Code	Course Name	Category	L	T	P	Credit	Year of Introduction
MA0M40A	FUNCTIONAL ANALYSIS	BTech Minors S7	3	1	0	4	2020

i) COURSE OVERVIEW: This course will cover the foundations of functional analysis in the context of basic real analysis, Metric spaces, Banach spaces and Hilbert spaces. Students learn various types of distances and associated results in these spaces. The important notion of linear functionals and duality will be developed in Banach space. An introduction to the concept of orthonormal sequences in Hilbert spaces enables them to efficiently handle with a variety of applications in engineering problems.

Prerequisite: Basic knowledge in set theory and linear algebra

COURSE OUTCOMES

After the completion of the course, the student will be able to:

Course Outcomes	Description	Learning Level
CO 1	Explain the concept and analytical properties of the real number system	Understand
CO 2	Illustrate the concept of metric space and discuss the properties interior, closure, denseness and separability in a metric space	Understand
CO 3	Explain the concepts of Cauchy sequence, completeness and Banach spaces and apply these concepts to metric and Banach spaces	Apply
CO 4	Demonstrate the concepts of linear operator, linear functional, dual basis and dual space of normed linear spaces	Understand
CO 5	Explain the notions of inner product and Hilbert space and apply the tools to construct orthonormal sequences in Hilbert spaces	Apply

ii) SYLLABUS

Module 1 (Real Analysis) Denumerable set, Countable set, Supremum and Infimum of a set, Sequence of real numbers, Convergent and Divergent sequence, Limit, Bounded sequence, Monotone sequence, Monotone convergence Theorem(without proof), Subsequence, Bolzano-Weierstrass theorem(without proof) , Cauchy sequence, Cauchy convergence criterion, Sequence of functions, Pointwise convergence, Uniform convergence, Uniform norm

Module 2 (Metric Space) Discrete space, Sequence space, Subspace, Holder inequality (without proof), Cauchy- Schwarz inequality (without proof), Minkowski inequality (without proof), Open set, Closed set, Neighbourhood, Interior, Continuous function, Accumulation point, Closure, Dense set, Separable space, Convergence of sequence, Limit, Bounded sequence.

Module 3 (Complete Metric Space and Normed space) Cauchy sequence in a metric space, Complete Metric Space, Completeness of , Convergent Sequence space, , Examples of incomplete metric spaces, Vector space with examples, Normed space, Banach space: , Metric induced by norm, Examples of incomplete normed spaces.

Module 4 (Space of Functionals and Operators) Properties of Normed Spaces ,Subspaces , Closed subspace, Schauder basis, Linear Operator, Range , Null space, Bounded Linear Operator, Norm of an operator, Linear operator on a finite dimensional space, Continuous linear operator, Relation between bounded and continuous operators, Linear functional, bounded linear functional, Algebraic dual space, Dual basis, Space B(X,Y) , Completeness of B(X,Y)(without proof), Dual space , Examples of dual space

Module - 5 (Hilbert Spaces) Inner Product Space, Hilbert Space, Parallelogram equality, Orthogonality, Examples of Hilbert Spaces ,Examples of Non-Hilbert spaces , Polarization identity, Further properties of inner product spaces - Schwartz inequality, Triangle inequality, Continuity of inner product, Subspace of an inner product space and Hilbert Space, Subspace Theorem, Convex set, Minimizing vector Theorem (without proof), Orthogonality Lemma (without proof), Direct sum, Orthogonal complement, Direct sum Theorem, Orthogonal projection, Null space Lemma, Closed subspace Lemma, Dense set Lemma, Orthonormal sets and sequences, Examples and properties, Bessel inequality, Gram-Schmidt process (without proof).

(iv) TEXT BOOKS

1. Robert G. Bartle and Donald R. Sherbert, Introduction to Real Analysis, JohnWiley & Sons, Inc., 4th Edition, 2011.
2. Erwin Kreyszig, Introductory Functional Analysis with Applications, John Wiley & Sons (Asia) Pte Ltd.

v) OTHER REFERENCES

1. Ajit Kumar, S. Kumaresan, A Basic Course in Real Analysis, CRC Press.
2. Herbert S. Gaskill, P P Narayanaswami, Elements of Real Analysis, Pearson.
3. Hiroyuki Shima, Functional Analysis for Physics and Engineering – An introduction, CRC Press, Taylor & Francis Group.
4. Balmohan V. Limaye, Linear Functional Analysis for Scientists and Engineers, Springer, Singapore, 2016.
5. Rabindranath Sen, A First Course in Functional Analysis- Theory and Applications, Anthem Press - An imprint of Wimbledon Publishing Company.
6. M. Tamban Nair, Functional Analysis- A first course, Prentice Hall of India Pvt. Ltd.

(v) COURSE PLAN

Module	Contents	No. of hours
I	Real Analysis -Denumerable set ,Countable set, Supremum and Infimum of a set, Sequence of real numbers, Convergent sequence, Limit , Divergent sequence, Bounded sequence, Monotone sequence, Monotone convergence Theorem(without proof), Subsequence, Bolzano-Weierstrass theorem(without proof), Cauchy sequence, Cauchy convergence criterion, Sequence of functions, Pointwise convergence, Uniform convergence, Uniform norm	12
II	Metric Space-Metric Space: $\mathbb{R}^n$, $\mathbb{C}^n$, l^∞,C[a, b], Discrete space, Sequence space, Space of bounded functions B(A), Subspace, Holder inequality (without proof), Cauchy- Schwarz inequality (without proof), Minkowski inequality (without proof), Open set, Closed set, Neighbourhood, Interior, Continuous function, Accumulation point, Closure, Dense set, Separable space, Convergence of sequence, Limit, Bounded sequence	12
III	Complete Metric Space and Normed space-Cauchy sequence, Complete Metric Space, Completeness of $\mathbb{R}^n$, $\mathbb{C}^n$, Completeness of l^∞, C[a, b], Completeness of Convergent Sequence space, l^p, Examples of incomplete metric spaces, Vector space with examples, Normed space, Banach space: $\mathbb{R}^n$, $\mathbb{C}^n$, l^∞,C[a, b], Metric induced by norm, Examples of incomplete normed spaces.	12
IV	Space of Functionals and Operators - Properties of Normed Spaces ,Subspaces, Closed subspace, Schauder basis, Linear Operator, Range , Null space, Bounded Linear Operator, Norm of an operator, Linear operator on a finite dimensional space, Continuous linear operator, Relation between bounded and continuous operators, Linear functional, bounded linear functional, Algebraic dual space, Dual basis, Space B(X,Y) , Completeness of B(X,Y)(without proof), Dual space , Examples of dual space.	12
V	Hilbert Space - Inner Product Space, Hilbert Space, Parallelogram equality, Orthogonality, Examples of Hilbert Spaces: , , $\mathbb{R}^n$ $\mathbb{C}^n$ l^2, Examples of Non-Hilbert spaces-l^p with $p\neq 2$,C[a,b] , Polarization identity, Schwartz inequality, Triangle inequality, Continuity of Inner product, Subspace of an inner product space and Hilbert Space, Subspace Theorem, Convex set, Minimizing vector Theorem (without proof), Orthogonality Lemma (without proof), Direct sum, Orthogonal complement. Direct Sum Theorem, Orthogonal Projection, Null space Lemma, Closed subspace Lemma, Dense set Lemma, Orthonormal sets and sequences, Examples and properties of Orthonormal sets, Bessel inequality, Gram-Schmidt process (without proof)	12
	Total hours	**60**

i) ASSESSMENT PATTERN

Bloom's Taxonomy Level	Continuous Assessment Exams (Marks)		End Semester Exam (Marks)
	CA Exam I	CA Exam II	
Remember	10	10	20
Understand	20	20	40
Apply	20	20	40
Analyse			
Evaluate			
Create			

ii) CONTINOUS ASSESSMENT EVALUATION PATTERN

Attendance	:	**10 marks**
CA Exams (2 numbers)	:	**25 marks**
Assignment/Project/Case study etc.	:	**15 marks**
Total	:	**50 marks**

iii) CONTINUOUS ASSESSMENT EXAMINATION PATTERN

- **Two tests of 50 marks** each (half the syllabus to be covered in each exam – 2 ½ modules)
- Duration – **2 hours**

iv) END SEMESTER EXAMINATION PATTERN

Duration – **3 hours**

Total marks -**100**