

Course Code	Course Name	Category	L	T	P	Credit	Year of Introduction
MA0M30B	ALGEBRA AND NUMBER THEORY	BTech Minors S6	3	1	0	4	2020

### i) COURSE OVERVIEW:

This is an introductory course in algebra and number theory with special emphasis on applications including RSA, prime factorization and the interplay between rings and numbers. **Prerequisite:** A basic understanding of set theory and logic..

### ii) COURSE OUTCOMES

After the completion of the course, the student will be able to:

Course Outcomes	Description	Learning Level
CO 1	Solve number theoretic problems by applying the concept and properties of natural numbers and applications of division algorithm and related results	Apply
CO 2	Utilise the concepts and properties learned about prime numbers and basic factorisation algorithms to solve number theoretic problems	Apply
CO 3	Solve algebraic problems using the concepts and properties of groups and group structures	Apply
CO 4	Utilise the concept, properties and applications of cyclic groups, permutations and symmetric groups to solve algebraic problems	Apply
CO 5	Solve algebraic problems using the concept, properties and applications of rings and ring structures	Apply

### iii) SYLLABUS

Module 1 (Elementary Number Theory) Division with remainder, congruences, greatest common divisor, Euclidean algorithm, Chinese remainder theorem, Euler's theorem

Module 2 (Prime Numbers) Prime Numbers- basic results, unique factorisation, computing Euler function, RSA explained, Fermat's little theorem, pseudoprimes, Algorithms for prime factorisation Fermat's and Fermat-Kraitchik algorithms (evaluation only), Quadratic residues.

Module 3 (Introduction to Groups) Groups- Definition- basic properties and examples, subgroups and cosets, normal subgroups, group homomorphisms. Isomorphism theorem

Module 4 (Further topics in Group theory) Order of a group element, Cyclic groups, symmetric groups, cycles, simple transpositions and bubble sort, alternating groups.

Module 5 (Ring Theory) Rings- Definition, ideals, principal ideal domain, Quotient rings, Prime and maximal ideals, Ring homomorphisms, unique factorisation domain, irreducible and prime elements, Euclidean domain.

#### (iv) TEXT BOOKS

Niels Lauritzen, “Concrete Abstract Algebra”, Cambridge University Press, 2003

#### OTHER REFERENCES

1. David M Burton, “Elementary Number Theory”, 7th edition, McGraw Hill, 2011
2. John B Fraleigh, “A first course in Abstract Algebra”. 7th edition, Pearson Education India, 2013
3. Joseph A Gallian, “Contemporary Abstract Algebra”, 9th edition, Cengage Learning India Pvt. Ltd

#### COURSE PLAN

Module	Contents	No. of hours
I	<b>Elementary Number Theory</b> -Division with remainder -Congruence- Properties of Congruence- Greatest Common divisor -Euclidean algorithm- Relatively prime numbers -Chinese Remainder Theorem- Euler’s Theorem	12
II	<b>Prime Numbers</b> - Basic Results- unique factorization- Computing $\phi$ -function- RSA explained- Fermat’s Little theorem, Pseudoprimes- Factorisation algorithms- Fermat’s algorithm- Fermat-Kraitchik algorithm- Quadratic residue- Quadratic residue applications	12
III	<b>Introduction to Groups</b> - DeSinition- Basic Properties- Examples- Subgroups- Cosets- Normal Subgroups- Quotient Groups- Group homomorphisms- Isomorphism theorem	12
IV	<b>Further topics in Group Theory</b> - Order of a group element- Cyclic Groups- Properties- Symmetric groups- Cycles- Properties- Simple transpositions- Bubble sort- Alternating groups.	12
V	<b>Ring Theory</b> - DeSinition, basic properties, ideals- Quotient rings- Prime and Maximal ideals- Ring homomorphisms,- Unique factorization- Irreducible elements- prime elements- Euclidean domain	12
	<b>Total hours</b>	<b>60</b>

**i) ASSESSMENT PATTERN**

<b>Bloom's Taxonomy Level</b>	<b>Continuous Assessment Exams (Marks)</b>		<b>End Semester Exam (Marks)</b>
	<b>CA Exam I</b>	<b>CA Exam II</b>	
Remember	10	10	20
Understand	20	20	40
Apply	20	20	40
Analyse			
Evaluate			
Create			

**ii) CONTINUOUS ASSESSMENT EVALUATION PATTERN**

Attendance	:	<b>10 marks</b>
CA Exams (2 numbers)	:	<b>25 marks</b>
Assignment/Project/Case study etc.	:	<b>15 marks</b>
<b>Total</b>	:	<b>50 marks</b>

**iii) CONTINUOUS ASSESSMENT EXAMINATION PATTERN**

- **Two tests of 50 marks** each (half the syllabus to be covered in each exam – 2 ½ modules)
- Duration – **2 hours**

**iv) END SEMESTER EXAMINATION PATTERN**

Duration – **3 hours**

Total marks -**100**